AP Physics **Vectors – A Guideline For Motion**

Introduction: You deal with scalar quantities in many aspects of your everyday activities. For example, you know that 2 liters plus 2 liters is 4 liters. The concept of volume has no other dimensions associated with it. Scalar quantities are not affected by anything other than their magnitude or sizes. Scalar quantities are added according to the rules of ordinary arithmetic.

You have probably encountered vector quantities in your everyday activities but may not have been aware that you were dealing with a vector quantity. Things that need more than one number to describe them are vector quantities. Vector quantities are entirely different from scalar quantities. The vector quantities (displacement, velocity, and acceleration), which you have encountered in this class so far, have had magnitude and direction. Force is another vector quantity we will study. The units for force are pounds in the English system and newtons in the international system. The sum of two vector quantities depends as much on their directions as their magnitudes. The sum of a 12 N force plus a 12 N force can equal from zero to 24 N depending on their directions.

In physics we often deal with vector quantities as well as scalar quantities. An understanding of both is fundamental to an understanding of the basic principles of physics.

Suppose you are not going to be a physicist. Why should you be concerned about the difference between scalars and vectors? L.C. Epstein in his book, Thinking Physics, answers the question this way. Because many people, who are not physicists, for example bureaucrats and business people, are asked to classify things, categorize things, or set up measuring schemes. These people frequently try to put things on a 1 – 10 or A – F scalar scale without first stopping to think about what they are classifying. Sometimes doing this really messes up what they are trying to do.

For example, the popular measure of intelligence is related as one number called IQ. That implies that intelligence is a scalar. But is intelligence really a scalar? Some people have good memories but can't reason. Some people learn quickly and forget quickly (crammers). Intelligence depends on many things like ability to learn, ability to remember, ability to reason, etc. So intelligence is a vector, not a scalar! That is a vital difference and the failure to recognize it has hurt thousands of people. So physicist or not, you had better get the idea of vector and scalar straight in your mind.

Performance Objectives: Upon completion of the readings and activities in this unit and when asked to respond either orally or on a written test, you will:

- Clearly demonstrate an understanding of the difference between a vector and a scalar quantity. Give examples of each.
- Distinguish between distance and displacement – speed and velocity.
- Show an ability to add any number of vectors by the graphical tip–to–tail method. Recognize the commutative law as applied to vectors.
- Be able to apply elementary geometry and trigonometry to vector algebra.
- Recognize the difference between a drawing of the actual situation and a vector diagram for the head–to–tail addition.
- Recognize the independence of vector quantities. Accurately interpret vectors jointly and separately.
- State the requirement for equilibrium. State the meaning of equilibrant. Distinguish between equilibrant and resultant.
- Resolve any vector into its perpendicular components graphically and trigonometrically. Use the method of rectangular resolution to add multiple vectors.
- Resolve graphically and trigonometrically a weight vector into components perpendicular and parallel to an inclined plane. Recognize that steeper inclines produce a lower normal force.

Textbook Reference: Serway & Jewett Physics for Scientists and Engineers: Chapter 3
Tipler Physics: Chapter 1, Section 7

"A number alone is, however, insufficient for describing some physical concepts. The recognition of this fact marked a distinct advance in scientific investigation."
-Albert Einstein (1879-1956) and Leopold Infield (1898-1968)

Introduction: A vector, which is directed line segment, can be used to represent a vector quantity. The length of the vector represents the magnitude and the direction shows the direction of the vector quantity. The vector equation for adding two vectors looks like an algebraic equation, but it is not. Rules for vector addition must be followed. Vectors can be added geometrically by drawing the vectors to scale in the appropriate direction and connecting them tip-to-tail. The sum of two or more vectors is called the resultant. The resultant is drawn from the tail of the first vector to the tip of the last added vector. When you add two vectors, the two added vectors and the resultant form a closed triangle.

Recall from your geometry and/or trigonometry classes that you can find the length of the third side of a triangle given the other two sides and the angle between these sides by using Pythagorean theorem or the law of cosines. Solving a vector equation algebraically usually means using one of these procedures, not just adding the magnitudes of the vectors. This gets confusing. Follow the examples given in class very carefully.

Exercises and Problems:

Draw vector diagrams to solve problems 1-7. Use a protractor, a sharp pencil, and a metric ruler.

1. A plane flying due north, at 100.0 m/s is blown due west at 50.0 m/s by a strong wind. Find the plane's resultant velocity. *112 m/s 27° W of N*

2. A hiker leaves camp and walks 10.0 km due north. The hiker then walks 10.0 km due east. a.) What distance does he walk? b.) Determine his total displacement from the starting point.
20 km 14 km 45° N of E

3. An airplane flies due west at 120.0 km/h. At the same time, the wind blows it from the north at 40.0 km/h. What is the plane's resultant velocity?
126.5 km/h @ 18° S of W

4. Two soccer players kick the ball at exactly the same time. One player's foot exerts a force of 60.0 N south. The other's foot exerts a force of 80.0 N east. Find the magnitude and direction of the resultant force on the ball. *100 N 37° S of E*

5. A weather team releases a weather balloon. The balloon's buoyancy accelerates it straight up at 15 m/s^2. A wind accelerates it horizontally at 6.5 m/s^2. Find the magnitude and direction (with reference to the horizontal) of the resultant acceleration.
16.3 m/s^2 *67° wrt horizontal*

6. What is the vector sun of a 65 N force acting due east and 90.0 N force acting due west? *25 N West*

7. A plane flies due north at 200.0 km/h. A wind blows it due east at 50.0 km/h. Find the magnitude and direction of the plane's resultant velocity.
206 km/h 14° E of N

(Solve 1 to 7 using Pythagorean Theorem and trig. functions.)

8. A salesperson leaves the office and drives 20.0 km due north along a straight highway. A turn is made onto a highway that leads in a direction 30.0° north of east. The driver continues on the highway for a distance of 62 km and then stops. What is the totally displacement of the sales person from the office? *74 km 43° N of E*

9. Two forces of 60.0 N each act concurrently on point P. Determine the magnitude of the resultant force acting on point P when the angle between the forces is as follows: a.) 0° b.) 30.0° c.) 60.0° d.) 90.0° e.) 135° f.) 180°
120 N 115N 104 N 85 N 46 N 0N

10. In problem 9, what happens to the resultant of two forces as the angle between them increases?

11. Determine the magnitude of the resultant of a 40.0 N force and 70.0 N force acting concurrently when the angle between them is: a.) 0° b.) 45° c.) 90.0° d.) 150° e.) 180°
110 N 102 N 81 N 41 N 30 N

12. An airplane flies at 150 km/h and heads 30.0° south of east. A 50.0 km/h wind blows in the direction 25° west of south. What is the resultant velocity of the plane? *162 km/h @ 48° S of E*

13. A 60.0 N force acting 30.0° east of north and a second 60.0 N force acting in the direction 60.0° east of north are concurrent forces. Determine the resultant force. *116 N @ 45° E of N*

14. A 60.0 N force acts 45° west of south. An 80.0 N force acts 45° north of west. The two forces act on the same point. Find the magnitude and direction of their resultant. *100 N @ 8° N of W*

15. A 30.0 N force acting due north and a 40 N force acting 35° east of north act concurrently on point P. Find the magnitude and direction of their resultant. *67 N @ 17° E of N*

Solve 8-15 again, this time use the Law of Cosine and the Law of Sines to solve them.)

Perpendicular components of vectors:

A component of a vector is its effective value in any given direction. A vector may be considered as the resultant of two or more component vectors. It is customary and most useful to resolve a vector into its perpendicular components (This is sometimes called rectangular or vector resolution).

16. A heavy box is pulled across a wooden floor with a rope. The rope forms an angle of 60.0° with the floor. A tension of 80.0 N is maintained on the rope. What force actually is pulling the box across the floor? *40.0 N*

17. The rope in the previous problem is lowered until it forms an angle of 30.0° with the floor. A force of 80.0 N is maintained on the rope. What force pulls the box across the floor? *69 N*

18. An airplane flies 30.0° north of west at 500.0 km/h. At what speed is the plane moving a.) north? b.) west? *250 km/h* *433 km/h*

19. A ship sails from Norfolk harbor. It maintains a direction of 45° north of east for a distance of 100.0 km. How many kilometers north and east has the ship progressed from Norfolk? *71 km*

20. A wind with a velocity of 40.0 km/h blows 30.0° north of east. What is the north component of the wind's velocity? What is the east component of the wind's velocity? *20 km/h North*

21. To cross a stream, which flows west to east, a boat steers at an angle of 60.0° north of west. The boat travels at 8.0 km/h in still water. What is the speed of the current and the boat's speed relative to the shore if a person on shore sees the boat come straight across the stream?
Current is 4km/h *Boat is 6.9 km/h*

22. A river flows due south. A riverboat pilot heads the boat 27° north of west and is able to go straight across the river at 6.0 m/s. a.) What is the speed of the current? b.) What is the speed of the boat?
3.1 m/s *6.7 m/s*

23. A lawn mower is pushed with a force of 70.0 N applied to the handle. Find the horizontal component of this force when the handle is held at an angle with the lawn of a.) 60.0° b.) 45.0° c.)30.0° *35 N* *50 N* *61 N*

24. A water skier is towed by a speedboat. The skier moves to one side of the boat in such a way that the tow rope forms an angle of 55° with the wake of the boat. The tension on the rope is 350 N. What component of this tension is helping the skier to go forward? What component of this tension is trying to return the skier to a position directly behind the boat? *201 N* *287 N*

Another Look at the Addition of Vectors:

Another method for adding vectors is to resolve the vectors to be added into perpendicular components. Add all the horizontal component vectors, then add all the vertical component vectors. This produces two vectors acting at a right angle. Pythagorean Theorem can be used to find the resultant. This method is especially useful when you are adding many vectors. Of course, if you prefer, you can still add several vectors graphically or using the law of cosines.

25. Add the following displacements: 8.0 m east, 5.0 m 30.0° north of east, 7.0 m 37° west of north, and 3.0 m south. *9.6 m* *32° N of E*

26. Find the resultant force of the three vectors acting concurrently on point P: 200.0 N 30.0° north of east, 300.0 N 45° north of west, and 155 N 55° south of west. *225 N* *55°* north of west

27. Find the resultant of the following displacements: 19 m east, 15 m 60.0° north of east, 16 m 45° north of west, 11 m 30.0° south of west, and 12 m south. *8.9 m* *50.0*° N of E

28. Find the resultant of the following set of forces: 200.0 N east, 300.0 N 60.0° north of east, 100.0 N 45° north of west, and 200.0 N south.
308 N *25° north of east*

Equilibrium:

When two or more forces act concurrently on an object and their vector sum is zero, the object is said to be in equilibrium. The word concurrent is a physics term which means not only at the same time but also that the lines of action of the forces acting on the object intersect.

Engineers are frequently concerned with the stability of structures and the forces that supporting girders must offset. To have stability the forces acting on one single object must be arranged in such a way as to produce no net force. The vector sum of the forces must be zero. A resultant force of zero is known as the first condition for equilibrium.

To find the equilibrant of two or more concurrent forces, first find the resultant of the forces. The equilibrant is a force equal in magnitude to the resultant, but opposite in direction.

29. Find the resultant and equilibrant in each of the following: a.) 60.0 N up; 20.0 N down; 5.0 N up.
45.0 N up 45.0 N down
b.) 10.0 N north; 30.0 N east; 10.0 N south.
30.0 N east 30.0 N west
c.) 50.0 N north; 10.0 N east; 20.0 N west; 40.0 N south. *14 N 45° N of W 14 N 45° S of E*

We all talk about the "force of gravity." This is a force exerted on all objects and each of us as well when we are in the earth's gravitational field. This force is called weight, W, of the body. Like all forces weight is a vector quantity and always directed toward the center of the earth. The weight vector is drawn perpendicular to the surface of the earth or perpendicular to the horizontal.

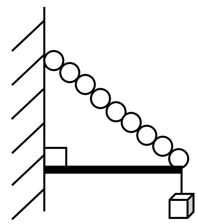
*As you work with, or read about forces in physics you will see another word used quite often. This word is **TENSION**. Tension is used to describe the force exerted by a rope, a rubber band, a fishing line, or a spring scale on two different bodies at the same time. For example, a string used to hang a picture. The string exerts a force on the nail in the wall and on the picture. Another way of looking at tension is to think of it as the force transferred from one body to another by the rope, string, etc. A rope or string, which is usually flexible, is rigid and said to be "under tension" when transferring a force.*

30. A picture is supported on a wall by a cord connected between two eyelets, one on either side of the picture, and hanging over a peg. At the peg, the cord on either side makes an angle of 30° with the horizontal. What is the tension in each cord if the picture weighs 20 N? *$T_1 = T_2 = 20$ N*

31. A 50.0 N picture is hung from a nail by two cords that make 45° angles with the horizontal. Find the tension in the cords. *35 N*

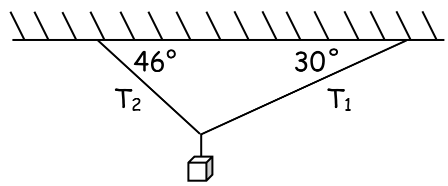
32. A 1000.0 N load hangs from the end of a boom as shown in the figure below. Find the pull (tension) of the chain and the push (compression) of the boom on the point from which the load is suspended - if the angle between the chain and the boom is 45˚.
boom 1000 N chain 1414 N

Use the following figures for 32 & 33…

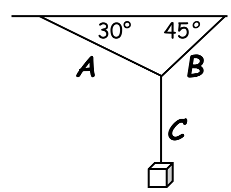


33. A 490 N weight hangs from the end of a horizontal strut protruding 2.5 m from a vertical wall. A tie rod connects the outer end of the strut to a point on the wall 1.5 m above the strut. What are the compression in the strut and the tension in the tie rod? *31° 815 N 951 N*

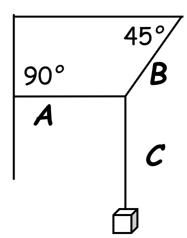
34. A weight of 1470.0 N is suspended as shown in the figure below. Find the tension in the supporting cables. *$T_1 = 1050$ N $T_2 = 1313$ N*



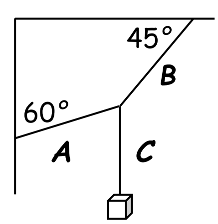
35. Find the tension in each cord in the figures below, if the weight of the suspended body is 200 N.



(a)



(b)



(c)

a.) $T_A = 146$ N
$T_B = 179$ N
b.) $T_A = 200$ N
$T_B = 283$ N
c.) $T_A = 546$ N
$T_B = 669$ N

36. A weight of 490 N hangs from a vertical 5.0 m rope. The mass on the rope is then pulled aside 3.0 m by a horizontal force. Find the horizontal force and the tension in the rope. Draw separate diagrams of the hanging weight and of the concurrent forces.
$T = 612$ N $F_H = 368$ N

37. A 500 kg mass suspended from a 3.00 m cord is pulled to one side by a horizontal force F_H so that the cord makes an angle of 30.0° with the vertical. What is the value of the force F_H? What is the tension in the cord? *F_H =2829 N* *T =5658 N*

38. A rope attached to the front end of a stalled automobile is fastened to a tree 20.0 m away. A person grasps the rope at the midpoint and pushes it in a direction perpendicular to itself. The person is able to move the rope 0.30 m. If the person was applying a force of 80.0 N to the rope, what force is exerted on the car by the rope?
from 1333 N to 1348 N depending on rounding

Please Note: *Vectors – when added – produce a resultant vector, but never alter each other's values! Each vector remains completely independent of all other vectors! The resultant merely indicates the combined effect of the vectors.*

Vector Difference:

It is sometimes necessary to subtract one vector from another. The process of subtracting one algebraic quantity from another is equivalent to adding the negative of the quantity to be subtracted, that is, a – b = a + (-b). Similarly, the negative **-B** of a vector quantity **B** is defined as a vector of the same length (or magnitude) but opposite in direction. Then the vector operation **A - B** is defined as the vector sum of **A** and **(-B)**. That is **A - B = A + (-B).** Vector subtraction is used frequently in connection with velocities and accelerations. Recall that:
$\Delta \boldsymbol{v} = \boldsymbol{v_f} - \boldsymbol{v_i}$.

39. A motorboat heads due east at 16 m/s across a river that flows due south at 9.0 m/s. a.) What is the resultant velocity of the boat? b.) If the river is 136 m wide, how long does it take the motorboat to reach the other side? c.) How far downstream is the boat when it reached the other side of the river?

40. A motorboat travels at 40.0 m/s. It heads straight across a river 320 m wide. a.) If the water flows at the rate of 8.0 m/s. What is the boat's velocity with respect to the shore? b.) How long does it take the boat to reach the opposite side?
41 m/s *8.0 sec*

41. A boat heads directly across a river 40.0 m wide at 8.0 m/s. The current is flowing at 3.8 m/s. a.) What is the resultant velocity of the boat? b.) How long does it take the boat to cross the river? c.) How far downstream is the boat when it reaches the other side? *8.9 m/s* *5.0 sec* *19 m*

42. An airplane is to maintain a velocity of 475 km/hr in a northeasterly direction. If the wind velocity is 50.0 km/hr southeast, what should be the magnitude and direction of the velocity of the airplane in order to offset the effect of the wind?
478 km/hr *39° E of N*

43. An airplane's destination is 200.0 miles east of its starting point. The wind is from the northwest at 30.0 miles/hr. The pilot wishes to arrive at his destination in forty minutes. What should be his heading and airspeed? *280 mi/hr* *4° N of E*

44. General Lee Friendly is to fly an airplane into enemy territory, 115 kilometers directly north of his present position. His plane travels at 85 km/hr when at full throttle and is to cross over the enemy line in precisely 1.8 hours. The wind is blowing from the west at 55 km/hr. At what angle relative to the north direction should he aim his plane for this to work out? Is his aircraft capable of getting him to his destination on time? *40.7° W of N* *YES!*

45. A truck is traveling east at 30 m/s. It makes a turn and 10.0 s later it is traveling 40 m/s south. What is the average acceleration while making the turn? *5.0 m/s^2 @ 37˚ west of south*

46. What is the change in velocity of a jogger who is initially jogging at 20 m/s @ 37˚ north of east, then slows down to 7 m/s and heads west? b) If the jogger slows down and changes direction in 10 seconds, what is his acceleration?
26 m/s @ 28˚ south of west *2.6 m/s^2 @ same*

Gravitational Force and Inclined Planes:

The gravitational attraction of the each acting on an object, its weight, W, is directed toward the center of the earth. This means that its weight, W, must act perpendicular to the surface of the earth, or as we say perpendicular to the horizontal. When an object is on an inclined plane, the plane prevents the weight, W, from acting perpendicular to the horizontal. Instead, the plane causes the weight, W, to be resolved into two components. One component, $F_{perpendicular}$, acts perpendicular to and into the inclined plane. The second component, $F_{parallel}$, acts parallel to and down the inclined plane. As the incline becomes steeper, the component of the weight acting down on the incline becomes greater. The component of the weight acting perpendicular to *the inclined plane becomes less. As a result, you find smooth inclines treacherous while walking particularly if they are steep. The normal force – the force that the inclined plane places on whatever object may be sitting on it – is equal in magnitude but opposite in direction to the perpendicular component of the object's weight. If the incline is steep enough, you may find yourself forced to run down its length when you had no intention of running. The force down the incline,* $F_{parallel}$, *accelerates you against your will.*

47. A 500.0 N trunk is placed on an incline plane that forms a 30.0° angle with the horizontal. a.) Calculate the values of F_N and $F_{||}$. b.) Calculate the values for the components parallel and perpendicular to the inclined plane when the angle is increased to 60.0°. c.) When the angle of an inclined increases, how do the force components acting on the trunk change? *433 N 250 N*

48. An automobile weighing 12,000.0 N is parked on a 37° slope. a.) What force tends to cause the auto to roll down the hill? b.) What is the normal force between the auto and the hill?
7222 N 9584 N

49. A box resting on a 40° inclined plane experiences a normal force of 6000 N. Find the weight of the box and the force parallel to the inclined plane. *7832 N 5034 N*

50. A physics student on a steep hill finds himself accelerated down the hill by a force of 640 N. If the student weighs 800 N, how steep is the hill, that is, what is the angle of incline? *53°*

Appendix:

Reconcile the physics concepts you learned in this unit with the vector notation you learned in your math class. Learn to use your calculator to speed your vector calculations.

When you resolve a vector into its perpendicular components you are actually converting from polar notation to rectangular notation. Most scientific calculators will do this very quickly. $(r,\theta) \rightarrow (x,y)$.

Unit Vector Notation:

$\boldsymbol{\hat{\imath}}$, $\boldsymbol{\hat{\jmath}}$ and $\boldsymbol{\hat{k}}$ are unit vectors in specific perpendicular directions. The magnitude for each vector is one. The $\boldsymbol{\hat{\imath}}$ direction is along the x-axis, the $\boldsymbol{\hat{\jmath}}$ direction is along the y-axis and the $\boldsymbol{\hat{k}}$ direction is along the z-axis. The expression, $5\hat{\imath}$ m/s, is another way of representing the velocity of 5 m/s in the positive x direction. The velocity vector, $\boldsymbol{v = 3\hat{\imath} + 4\hat{\jmath}}$, where v is measured in m/s could represent an instantaneous velocity, an average velocity, or a constant velocity with an x-component of 3 m/s and a y-component of 4 m/s . From the components, you can calculate the magnitude and direction of v. The position vector (sometimes called the radius vector because it is radiating from the origin), $\boldsymbol{r}(t) = (3t)\hat{\imath} + (-4.9t^2)\hat{\jmath}$, tells you that position is a function of time and along the x-axis the position varies directly with time and along the y-axis the position varies with the square of time. The velocity vector, $\boldsymbol{v}(t) = dx/dt\ \hat{\imath} + dy/dt\ \hat{\jmath}$, gives the speeds in the x and y directions.

51. The position of a particle is given by the position vector, $\boldsymbol{r}(t) = (3t)\hat{\imath} + (-4.9t^2)\hat{\jmath}$, where r is in meters and t is in seconds.
a) What is its position at t = 2 s?
b) What is the instantaneous velocity of the particle?
c) What is the instantaneous velocity at t = 2 s?
d) What is the acceleration of the particle?
e) Sketch the position, velocity, and acceleration vectors at t = 2 s.